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Energy wheel effectiveness: part I—development of dimensionless groups

C.J. Simonson, R.W. Besant*

Department of Mechanical Engineering, University of Saskatchewan, 57 Campus Drive, Saskatoon, SK, Canada S7N 5A9

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Résumé

The fundamental dimensionless groups for air-to-air energy wheels that transfer both sensible heat and water vapor are derived from the governing non-linear and coupled heat and moisture transfer equations. These dimensionless groups for heat and moisture transfer are found to be functions of the operating temperature and humidity of the energy wheel. Unlike heat exchangers that transfer only sensible heat, the effectiveness of energy wheels is a function of the operating temperature and humidity as has been observed by several energy wheel manufacturers and researchers. The physical meaning of the dimensionless groups and the importance of the operating condition factor (H^*) are explained. The dimensionless groups are used in Part II to develop effectiveness correlations for energy wheels. © 1999 Elsevier Science Ltd. All rights reserved.

Nomenclature

A cross-sectional area of one tube [m^2]
A_s' heat and mass transfer surface area of one tube [m^2]
A_s heat and mass transfer surface area on the supply or exhaust side [m^2]
C constant describing the shape of the sorption curve
C^* ratio of the minimum to maximum heat capacity rate of the air streams
Cp specific heat [J (kg^{-1} K^{-1})]
Cr^* matrix heat (or moisture) capacity ratio on the supply or exhaust side
Crm^* matrix moisture capacity ratio on the supply or exhaust side
Cr_o^* overall matrix heat (or moisture) capacity ratio
Crm_o^* overall matrix moisture capacity ratio
D_h hydraulic diameter of one tube in the energy exchanger [m]
f general function
h convective heat transfer coefficient [W (m^{-2} K^{-1})]
$(hA_s)^*$ convective conductance ratio $(hA_s)_s/(hA_s)_e$
h_m convective mass transfer coefficient [m s^{-1}]
h_{fg} heat of vaporization [J kg^{-1}]
H total enthalpy per mass of dry air [J kg$_a^{-1}$]
H^* operating condition factor that represents the ratio of latent to sensible enthalpy differences between the inlets of the energy wheel
k thermal conductivity [W (m^{-1} K^{-1})]
L length of the heat exchanger [m]
Le Lewis number
$\dot{m}$ mass flow rate of dry air (unless specified by a specific subscript) [kg s^{-1}]
$\dot{m}'$ rate of phase change per unit length [kg (s^{-1} m^{-1})]
M total mass [kg]
N angular speed of the wheel [cycles s^{-1}]
NTU number of transfer units on the supply or exhaust side
NTU_o overall number of transfer units
p period of exposure per cycle for the supply or exhaust gas [s cycle^{-1}]
P pressure [Pa]
R specific gas constant [J (kg^{-1} K^{-1})]
t time [s]
t^* dimensionless time
T bulk temperature [K or °C]
u mass fraction of water in the desiccant [kg$_w$ kg$_d^{-1}$]
U mean air flow velocity in the tube [m s^{-1}]
W humidity ratio [kg$_w$ kg$_a^{-1}$]
Wm empirical coefficient used in the sorption isotherm describing the maximum moisture capacity of the desiccant [kg$_w$ kg$_d^{-1}$]

* Corresponding author. Fax: 001 306 966 5427

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x axial coordinate [m]
x^* dimensionless axial coordinate.

Greek symbols
α thermal diffusivity [$m^2\ s^{-1}$]
Δ difference between supply inlet and exhaust inlet conditions
ε effectiveness
ε_{cf} effectiveness of a counter flow recuperator
ε_l latent heat transfer (or moisture transfer) effectiveness
ε_s sensible heat transfer effectiveness
ε_t total enthalpy effectiveness
η fraction of the phase change energy that is delivered directly to the air
ρ density [$kg\ m^{-3}$]
ϕ relative humidity
χ general dependent variable.

Subscripts
a air
e exhaust side
d desiccant
dry dry properties
g total gas phase (air and water vapor)
ht dimensionless heat transfer group for energy wheels
i inlet
m matrix (including support material, desiccant and moisture)
min minimum
mt dimensionless moisture transfer group for energy wheels
o outlet or overall dimensionless group applying to the entire wheel
s supply side
sat saturation properties
v water vapor
w liquid water or frost.

1. Introduction

Heat exchangers, when combined with mass transfer, have somewhat more complex characteristics than devices that transfer only sensible energy. As a result, there is no simple design methodology available for regenerative rotary energy wheels which transfer both sensible heat and water vapor. Despite the lack of accepted design methods or effectiveness correlations, energy wheels are rapidly being introduced into HVAC designs because they can reduce cooling and heating loads in buildings [1, 2]. Energy wheels can also increase thermal comfort, while decreasing HVAC systems operating and capital costs [3–5]. Because energy wheels are beginning to be used more frequently, a simplified design methodology is needed to help HVAC engineers optimize designs for different climates and operating conditions.

Effectiveness is widely accepted as the best means to characterize and design most types of heat exchangers operating under a wide range of operating conditions where only the inlet properties are known [6, 7]. For sensible energy exchange, effectiveness (ε) is defined as the ratio of the actual heat transfer to the thermodynamic maximum heat transfer and therefore is restricted to values between 0 and 1. The ε–NTU is a favored design method because ε typically depends on two dimensionless groups for sensible recuperative heat exchangers (NTU and C^*) and four dimensionless groups for sensible regenerative heat exchangers (NTU_o, C^*, Cr_o^* and $(hA_s)^*$) [8–10]. The effect of $(hA_s)^*$ on effectiveness is negligible for the range of $0.25 \leqslant (hA_s)^* \leqslant 4$ and is neglected in this paper. These dimensionless groups are functions of the size and shape of the heat exchanger and the flow rate through the heat exchanger; but, for most designs, they are only weak functions of the operating temperatures of the heat exchanger when condensation and frosting are not significant. That is, for a given heat exchanger design with known flow rates, the effectiveness is essentially constant regardless of small changes in inlet fluid temperatures. This important characteristic of heat exchangers transferring sensible energy allows the designer to estimate energy savings over a wide range of operating temperatures without iterative calculation procedures.

By analogy with sensible heat exchangers, one might expect the effectiveness of a regenerative energy exchanger to be nearly constant for a range of inlet temperatures and humidities. However, recent experimental and theoretical evidence shows otherwise [11–14]. The effectiveness of energy wheels seems to be particularly affected by changes in the inlet humidities, but a recent correlation by Stiesch et al. [14] does not account for humidity effects nor does it give physical insight into why effectiveness varies as a function of the inlet operating conditions. This needs physical explanation. It is proposed that the key to understanding the link between operating conditions and effectiveness lies in the dimensionless groups derived from the governing equations.

This paper, Parts I and II, is concerned with several objectives. Firstly, new dimensionless parameters need to be developed to give insight into why effectiveness is so strongly dependent on the operating temperature and humidity. Using these parameters, new effectiveness correlations need to be developed for energy wheels that will allow for accurate designs. The new effectiveness correlations will help manufacturers and HVAC engineers to produce and select efficient energy wheels that provide greater life cycle cost savings.

2. Governing equations

The governing equations, presented in this section, use the coordinate system shown in Fig. 1, which contains a schematic of an energy wheel operating in a counter flow arrangement showing one of the tubes in detail. The governing equations are for simultaneous and coupled heat and moisture transfer in one tube as it rotates around the axis of the wheel. A typical operating condition would be warm humid supply air transferring energy and water vapor to the matrix and energy and moisture being transferred from the matrix to the exhaust air during the second half of the cycle.

The governing equations for simultaneous heat and moisture transfer in energy wheels, based on the assumptions listed in Table 1, are as follows [12, 15]:

Table 1
Assumption used in governing equations

1. Axial heat conduction and water vapor diffusion in the air are negligible.
2. Axial molecular diffusion and capillary motion of moisture within the desiccant are negligible.
3. There are no radial temperature or moisture content gradients in the matrix.
4. The half-plane and ends of each matrix tube are impermeable and adiabatic.
5. Hysteresis in the sorption isotherm for the desiccant coating is neglected and the heat of sorption is assumed constant and equal to the heat of vaporization.
6. The tubes that make up the rotary energy exchanger are identical with constant heat and mass transfer surface areas.
7. The matrix thermal and moisture properties (support material, desiccant and adsorbed water) are constant.

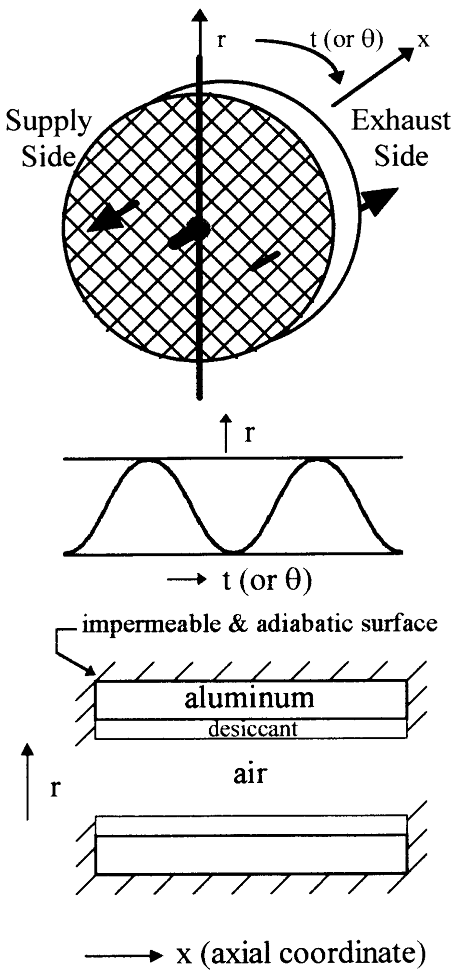

Fig. 1. Schematic of the rotary energy exchanger wheel showing (a) the entire wheel, (b) the tube geometry cross-section and (c) a side view of one of the tubes.

$$\rho_g Cp_g A_g \frac{\partial T_g}{\partial t} + U\rho_g Cp_g A_g \frac{\partial T_g}{\partial x} - \dot{m}' h_{fg} \eta + h\frac{A_s'}{L}(T_g - T_m) = 0 \quad (1)$$

$$\rho_m Cp_m A_m \frac{\partial T_m}{\partial t} - \dot{m}' h_{fg}(1-\eta) - \dot{m}' Cp_w (T_g - T_m) - h\frac{A_s'}{L}(T_g - T_m) = \frac{\partial}{\partial x}\left(k_m A_m \frac{\partial T_m}{\partial x}\right) \quad (2)$$

$$A_g \frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial x}(\rho_v U A_g) + \dot{m}' = 0 \quad (3)$$

$$\frac{\partial \rho_a}{\partial t} + \frac{\partial}{\partial x}(\rho_a U) = 0 \quad (4)$$

and

$$\dot{m}' = \rho_{d,dry} A_d \frac{\partial u}{\partial t} \quad (5)$$

where, during adsorption and desorption, the moisture transfer ($\dot{m}'$) can be calculated by

$$\dot{m}' = h_m \frac{A_s'}{L}(\rho_v - \rho_{v,m}). \quad (6)$$

During saturation conditions, the density of the water vapor is calculated using

$$\rho_v = \rho_{v,sat} = \frac{P_{v,sat}}{R_v T_g} \quad (7)$$

where $P_{v,sat}$ is the saturation vapor pressure and is a function of temperature only.

In the energy equations, the symbol η is the fraction of the phase change energy that is convected directly into the air. It can be estimated with

$$\eta = \frac{hD_h/\sqrt{\alpha_s}}{hD_h/\sqrt{\alpha_a} + k_d/\sqrt{\alpha_d}} \quad (8)$$

which is derived by Simonson and Besant [12]. When $\eta = 1$, all the energy of phase change is convected directly to the air and when $\eta = 0$, all the energy of phase change is conducted into the matrix. For $0 < \eta < 1$, some of the energy of phase change is convected into the air and the rest is conducted into the matrix. The value of η will not be constant for all energy wheels, but is expected to range between 0 and 0.1. η will also depend on the desiccant coating and the manufacturing process and the exact value of η needs to be determined experimentally. A more complete discussion of the value of η and its effect on the energy wheel performance is presented in the literature [12, 13]. Several property relations, boundary conditions and initial conditions are needed to complete the formulation of the problem; these can be found in Simonson and Besant [12].

3. Governing dimensionless groups

The method of deriving the governing dimensionless heat and moisture transfer groups for energy wheels from the governing equations follows the method presented by Shah [9]. Shah [9] shows that a set of simplified governing equations that govern the transfer and storage of sensible energy on the supply or exhaust side of a counterflow sensible regenerative heat exchanger can be approximated by

$$\frac{\partial T_g}{\partial x^*} = NTU(T_m - T_g) \tag{9}$$

and

$$\frac{\partial T_m}{\partial t^*} = \frac{NTU}{Cr^*}(T_g - T_m). \tag{10}$$

The form of equations (9) and (10) will be referred to as the standard dimensionless form. The dimensionless number of heat transfer units and matrix heat capacity on the supply and exhaust side are

$$NTU = \frac{hA'_s}{(\rho AUCp)_g} = \frac{hA_s}{(\dot{m}Cp)_g} \tag{11}$$

and

$$Cr^* = \frac{(\rho ACp)_m L}{(\rho AUCp)_g p} = \frac{(MCp)_m N}{(\dot{m}Cp)_g}. \tag{12}$$

Recognizing that the supply and exhaust sides of the heat exchanger may have different heat transfer coefficients or surface areas, a set of dimensionless groups which parallels those of a recuperator can be defined as

$$NTU_o = \frac{1}{(\dot{m}Cp)_{g,\min}}\left[\frac{1}{(hA_s)_s} + \frac{1}{(hA_s)_e}\right]^{-1}$$

$$\text{and} \quad Cr_o^* = \frac{(MCp)_m N}{(\dot{m}Cp)_{g,\min}}. \tag{13}$$

In the case of equal flow areas, heat transfer coefficients and mass flow rates on the hot and cold side, NTU_o and Cr_o^* become simply

$$NTU_o = \frac{hA_s}{2(\dot{m}Cp)_{g,\min}} = \frac{NTU}{2} \quad \text{and} \quad Cr_o^* = Cr^*. \tag{14}$$

Kays and London [10] and Shah [9] present the following empirical correlation, which demonstrates the effect of Cr_o^* and NTU_o on the effectiveness of a sensible heat regenerator:

$$\varepsilon = \varepsilon_{cf}\left[1 - \frac{1}{9(Cr_o^*)^{1.93}}\right] \tag{15a}$$

where

$$\varepsilon_{cf} = \frac{1 - \exp(-NTU_o(1-C^*))}{1 - C^* \exp(-NTU_o(1-C^*))} \tag{15b}$$

and C^* is the ratio of minimum to maximum heat capacity rate of the air streams. Equation (15) was obtained by correlating the numerical results of Lambertson [16] and Bahnke and Howard [17] and is recommended for $\varepsilon \leqslant 90\%$. Equation (15) agrees with the numerical data compiled in Kays and London [10] within 1% for $C^* = 1$ and the following parameter ranges: $2 < NTU_0 < 14$ for $Cr_o^* \geqslant 1.5$; $NTU_o \leqslant 20$ for $Cr_o^* \geqslant 2$; and a complete range of NTU_o for $Cr_o^* \geqslant 5$. As C^* decreases, the error in equation (15) increases with lower values of Cr_o^*. As Cr_o^* becomes large, the effectiveness approaches the effectiveness of a counterflow heat recuperator (ε_{cf}). Increasing NTU_o and Cr_o^* increases the effectiveness of a sensible rotary heat exchanger as is shown in Fig. 2.

3.1. Moisture transfer in energy wheels

Starting with the equations that govern the transfer [equation (3)] and storage [equation (5)] of moisture, the dimensionless groups that govern the moisture transfer process (i.e. NTU_{mt} and Cr_{mt}^*) are developed for balanced mass flow rates ($\dot{m}_s = \dot{m}_e$).

3.1.1. NTU_{mt}

Combining the equation for moisture transfer (3) with the rate of sorption phase change (6) gives

$$A_g \frac{\partial \rho_v}{\partial t} + \frac{\partial}{\partial x}(\rho_v U A_g) + h_m \frac{A'_s}{L}(\rho_v - \rho_{v,m}) = 0. \tag{16}$$

Introducing dimensionless variables for t and x as

$$x^* = \frac{x}{L} \tag{17a}$$

and

$$t^* = \frac{1}{p}\left(t - \frac{x}{U}\right) \tag{17b}$$

where the second term in t^* is small for small dwell times or carry over rates. The time and spatial derivatives can be written for any dependent variable, χ, as follows:

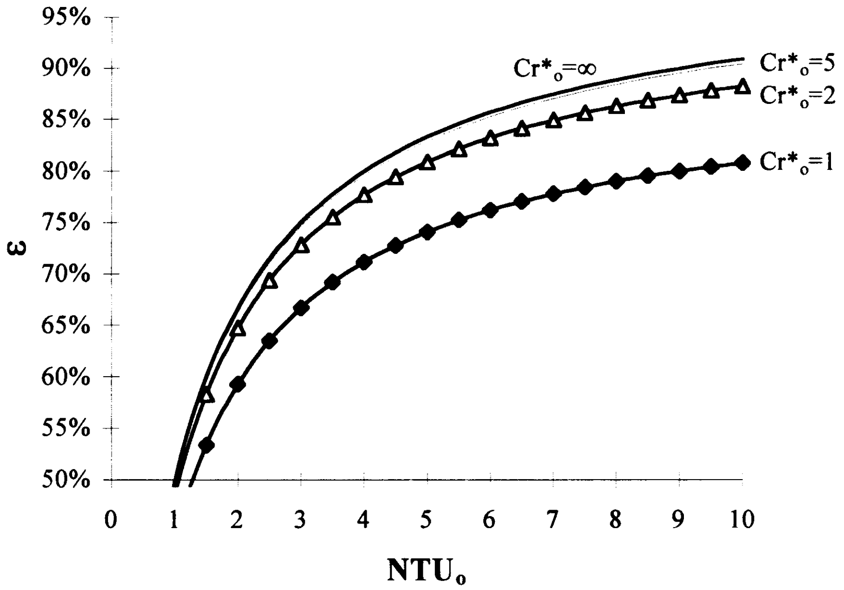

Fig. 2. Effectiveness of a counterflow rotary heat exchanger as a function of NTU_o and Cr^*_o for $C^* = 1$ [equation (15)].

$$\frac{\partial \chi}{\partial t} = \frac{\partial \chi}{\partial t^*}\frac{\partial t^*}{\partial t} = \frac{1}{p}\frac{\partial \chi}{\partial t^*} \tag{18a}$$

and

$$\frac{\partial \chi}{\partial x} = \frac{\partial \chi}{\partial x^*}\frac{\partial x^*}{\partial x} + \frac{\partial \chi}{\partial t^*}\frac{\partial t^*}{\partial x} = \frac{1}{L}\frac{\partial \chi}{\partial x^*} - \frac{1}{pU}\frac{\partial \chi}{\partial t^*}. \tag{18b}$$

It should be noted that the basic equations [i.e., equations (1)–(8)] and the above equations apply to the supply or exhaust side of the energy exchanger so that each variable could have a subscript s or e. The subscripts have been left out for clarity for the above transformations. Substituting equations (18) into equation (16) and rearranging yields

$$\frac{\partial \rho_v}{\partial x^*} = \frac{h_m A'_s}{A_g U}(\rho_{v,m} - \rho_v) + \frac{\rho_v}{U}\left(\frac{L}{pU}\frac{\partial U}{\partial t^*} - \frac{\partial U}{\partial x^*}\right). \tag{19}$$

If it is assumed that the density of dry air is constant or changes little within the energy wheel, the velocity gradient and time derivative can be neglected and equation (19) can be approximated by writing it in the standard dimensionless form similar to (9):

$$\frac{\partial \rho_v}{\partial x^*} = NTU_{mt}(\rho_{v,m} - \rho_v). \tag{20}$$

NTU_{mt} is the number of moisture transfer units on the supply or exhaust side of the energy wheel defined as

$$NTU_{mt} = \frac{h_m A'_s}{A_g U}. \tag{21}$$

If the Lewis number ($h/h_m \rho_g Cp_g$) is unity, which is a good approximation for simultaneous heat and water vapor transfer in air, NTU_{mt} can be rewritten as

$$NTU_{mt} = \frac{h}{\rho_g Cp_g}\frac{A'_s}{A_g U} = \frac{hA_s}{(\dot{m}Cp)_g} = NTU. \tag{22}$$

Equation (22) shows that for $Le = 1$, the number of moisture transfer units for an energy wheel is equal to the number of heat transfer units for a sensible rotary heat exchanger. This means that the number of moisture transfer units is independent of the operating humidity and temperature. If, however, the density of the air was not considered constant, NTU_{mt} would be slightly dependent on the operating conditions. Finally, it should be noted that NTU_{mt} on the supply side of the energy wheel will not always be equal to NTU_{mt} on the exhaust side, but these values will be nearly equal for most practical operating conditions.

3.1.2. Cr^*_{mt}

Equation (5) describes the storage of moisture in the desiccant coated matrix. Introducing the dimensionless independent variables (17), and substituting equation (6) into equation (5) gives

$$\frac{h_m A'_s}{L}(\rho_v - \rho_{v,m}) = \frac{\rho_{d,dry} A_d}{p}\frac{\partial u}{\partial t^*} = \frac{\rho_{d,dry} A_d}{p}\frac{\partial u}{\partial \rho_{v,m}}\frac{\partial \rho_{v,m}}{\partial t^*}. \tag{23}$$

Rearranging equation (23) gives an equation in the standard dimensionless form similar to (10):

$$\frac{\partial \rho_{v,m}}{\partial t^*} = \frac{NTU_{mt}}{Cr^*_{mt}}(\rho_v - \rho_{v,m}) \tag{24}$$

where

$$Cr^*_{\mathrm{mt}} = \frac{\rho_{\mathrm{d,dry}} A_{\mathrm{d}} L}{A_{\mathrm{g}} U p} \frac{\partial u}{\partial \rho_{\mathrm{v,m}}}. \quad (25)$$

Defining a general sorption curve as

$$u = \frac{Wm}{1 - C + C/\phi} = \frac{Wm}{1 - C + C/\phi_{\mathrm{m}}} \quad (26)$$

Cr^*_{mt} becomes

$$Cr^*_{\mathrm{mt}} = Crm^* \frac{\partial u}{\partial \phi} \frac{\phi}{W} \quad (27)$$

where $\phi_{\mathrm{m}} \approx \phi$, $\rho_{\mathrm{v,m}} \approx \rho_{\mathrm{v}}$ and

$$Crm^* = \frac{\rho_{\mathrm{d,dry}} A_{\mathrm{d}} L}{\rho_{\mathrm{a}} A_{\mathrm{g}} U p} = \frac{M_{\mathrm{d,dry}} N}{\dot{m}}. \quad (28)$$

Further simplification of equation (27) to include those parameters most familiar to an HVAC engineer (T and ϕ) by using the Clapeyron equation to represent the saturation vapor pressure and assuming standard atmospheric pressure (101 325 Pa) gives

$$Cr^*_{\mathrm{mt}} = (Crm^*) \frac{\partial u}{\partial \phi} \left(\frac{e^{\left(\frac{5294}{T}\right)}}{10^6} - 1.61\phi \right) \quad (29)$$

where T is in K. The second term in the brackets in equation (29) will generally have less than a 5% effect; thus, Cr^*_{mt} is a function of the slope of the sorption curve and the operating temperature and to a lesser extent the relative humidity.

3.2. Heat transfer

By performing a similar analysis as in the previous section, the governing dimensionless groups for heat transfer can be obtained.

3.2.1. NTU$_{ht}$

Starting with the equation that describes the transfer of heat between the air and the matrix [i.e., equation (1)] and introducing the dimensionless independent variables and equation (6) (which applies to sorption only) gives

$$\frac{\rho_{\mathrm{g}} C p_{\mathrm{g}} A_{\mathrm{g}}}{p} \frac{\partial T_{\mathrm{g}}}{\partial t^*} + U \rho_{\mathrm{g}} C p_{\mathrm{g}} A_{\mathrm{g}} \left(\frac{1}{L} \frac{\partial T_{\mathrm{g}}}{\partial x^*} - \frac{1}{pU} \frac{\partial T_{\mathrm{g}}}{\partial t^*} \right) - h_{\mathrm{m}} \frac{A'_{\mathrm{s}}}{L} (\rho_{\mathrm{v}} - \rho_{\mathrm{v,m}}) h_{\mathrm{fg}} \eta + h \frac{A'_{\mathrm{s}}}{L} (T_{\mathrm{g}} - T_{\mathrm{m}}) = 0. \quad (30)$$

Rearranging equation (30) yields

$$\frac{\partial T_{\mathrm{g}}}{\partial x^*} = \frac{h A'_{\mathrm{s}}}{U \rho_{\mathrm{g}} C p_{\mathrm{g}} A_{\mathrm{g}}} (T_{\mathrm{m}} - T_{\mathrm{g}}) - \frac{h_{\mathrm{m}} A'_{\mathrm{s}}}{U \rho_{\mathrm{g}} C p_{\mathrm{g}} A_{\mathrm{g}}} (\rho_{\mathrm{v,m}} - \rho_{\mathrm{v}}) h_{\mathrm{fg}} \eta. \quad (31)$$

Putting equation (31) in the standard dimensionless form of equation (9) gives

$$\frac{\partial T_{\mathrm{g}}}{\partial x^*} = NTU_{\mathrm{ht}} (T_{\mathrm{m}} - T_{\mathrm{g}}) \quad (32)$$

where

$$NTU_{\mathrm{ht}} = NTU \left[1 - \frac{h_{\mathrm{fg}} \eta}{\rho_{\mathrm{g}} C p_{\mathrm{g}}} \frac{(\rho_{\mathrm{v,m}} - \rho_{\mathrm{v}})}{T_{\mathrm{m}} - T_{\mathrm{g}}} \right] \quad (33)$$

because $Le = 1 = h/(h_{\mathrm{m}} \rho_{\mathrm{g}} C p_{\mathrm{g}})$. For NTU_{ht} to be as useful as a design tool, it needs to be calculated from known or determinable quantities. Clearly the ratio $(\rho_{\mathrm{v,m}} - \rho_{\mathrm{v}})/(T_{\mathrm{m}} - T_{\mathrm{g}})$ is not directly known to the designer, but can be related to the number of heat and moisture transfer units. Using equations (20) and (32) this unknown ratio can be expressed as

$$\frac{(\rho_{\mathrm{v,m}} - \rho_{\mathrm{v}})}{T_{\mathrm{m}} - T_{\mathrm{g}}} = \frac{NTU_{\mathrm{ht}}}{NTU_{\mathrm{mt}}} \frac{\frac{\partial \rho_{\mathrm{v}}}{\partial x^*}}{\frac{\partial T_{\mathrm{g}}}{\partial x^*}} \approx \frac{NTU_{\mathrm{ht}}}{NTU_{\mathrm{mt}}} \frac{\varepsilon_{\mathrm{l}} \Delta W}{\varepsilon_{\mathrm{s}} \Delta T_{\mathrm{g}}} \rho_{\mathrm{a}} \quad (34)$$

where $\varepsilon_{\mathrm{l}} = [(W_{\mathrm{i}} - W_{\mathrm{o}})_{\mathrm{s}}/(W_{\mathrm{s,i}} - W_{\mathrm{e,i}})]$, $\varepsilon_{\mathrm{s}} = [(T_{\mathrm{i}} - T_{\mathrm{o}})_{\mathrm{s}}/(T_{\mathrm{s,i}} - T_{\mathrm{e,i}})]$, $\Delta W = W_{\mathrm{s,i}} - W_{\mathrm{e,i}}$, $\Delta T_{\mathrm{g}} = T_{\mathrm{g_{s,i}}} - T_{\mathrm{g_{e,i}}}$ and ρ_{v} and T_{g} are assumed to be linear functions of x^*. Equation (34) can be inserted in equation (33) to give

$$NTU_{\mathrm{ht}} = NTU \left[1 - \frac{h_{\mathrm{fg}} \eta}{\rho_{\mathrm{g}} C p_{\mathrm{g}}} \frac{NTU_{\mathrm{ht}}}{NTU_{\mathrm{mt}}} \frac{\varepsilon_{\mathrm{l}}}{\varepsilon_{\mathrm{s}}} \frac{\Delta W}{\Delta T_{\mathrm{g}}} \rho_{\mathrm{a}} \right]. \quad (35)$$

Equation (35) can be further simplified by defining an operating factor as

$$H^* = \frac{h_{\mathrm{fg}}}{\rho_{\mathrm{g}} C p_{\mathrm{g}}} \rho_{\mathrm{a}} \frac{\Delta W}{\Delta T_{\mathrm{g}}} \approx 2500 \frac{\Delta W}{\Delta T_{\mathrm{g}}} \quad (36)$$

and using the fact that $NTU = NTU_{\mathrm{mt}}$ for $Le = 1$. The resulting dimensionless number of heat transfer units is

$$NTU_{\mathrm{ht}} = \frac{NTU}{1 + \eta \frac{\varepsilon_{\mathrm{l}}}{\varepsilon_{\mathrm{s}}} H^*}. \quad (37)$$

The number of heat transfer units for an energy wheel is a function of NTU (with no moisture transfer), the ratio of latent to sensible effectiveness and the operating conditions such that, as the fraction of phase change energy entering the air (η) goes to zero, only NTU is a factor. The operating condition factor (H^*) can in theory vary from $-\infty$ to $+\infty$, but varies typically from -6 to $+6$ for energy wheels used in HVAC applications.

The operating condition factor (H^*) is essentially a ratio of latent to sensible energy differences across the energy wheel. This enthalpy ratio can also be expressed in terms of the sensible heat ratio between two operating points which is familiar to HVAC engineers. The relationship is

$$H^* = \frac{\Delta H_{\mathrm{l}}}{\Delta H_{\mathrm{s}}} = \left(\frac{\Delta H_{\mathrm{s}}}{\Delta H_{\mathrm{t}}} \right)^{-1} - 1. \quad (38)$$

Therefore, H^* can be determined graphically by joining the inlet conditions on the psychrometric chart and determining the value of the sensible heat ratio ($\Delta H_{\mathrm{s}}/\Delta H_{\mathrm{t}}$) from the protractor and then calculating H^* from equation (38). For numerical calculations, H^* can be calculated from equation (36).

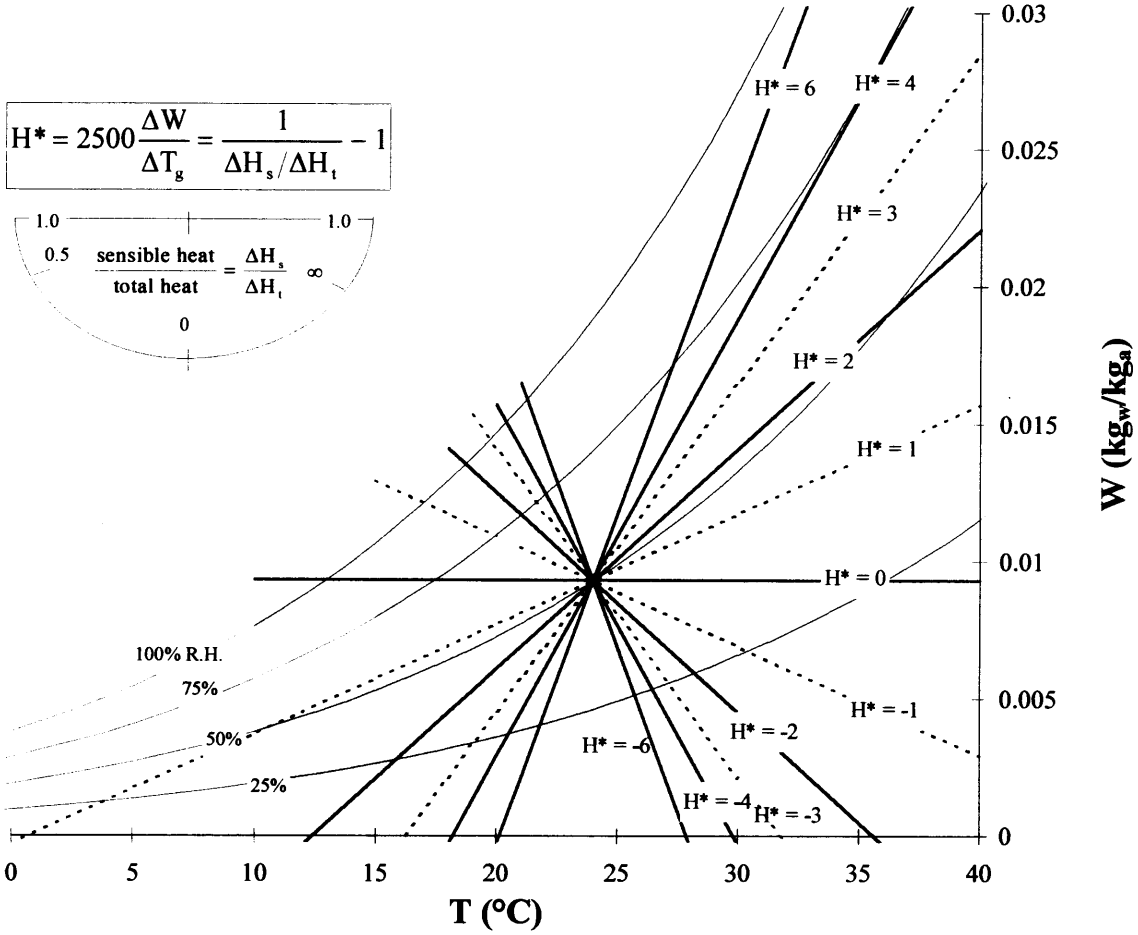

Fig. 3. Psychrometric chart showing the operating condition factor (H^*) for exhaust conditions of 24°C and 50% RH and different supply conditions.

Figure 3 shows lines of constant H^* for exhaust condition of 24°C and 50% RH and various supply conditions on the psychrometric chart. The value of η is typically less than 0.1 for energy wheels and the ratio of $\varepsilon_l/\varepsilon_s$ will tend to be near unity for practical applications. Therefore, the value of NTU_{ht} will tend to be between 0.5–3 times the value of NTU for a wheel where no moisture transfer occurs.

If H^* is positive (i.e. heat and moisture transfer in the same direction between the supply and exhaust air), the moisture transfer will decrease NTU_{ht} and the sensible effectiveness. On the other hand, if H^* is negative, the moisture transfer will increase NTU_{ht}. This can be explained by the fact that adsorption is exothermic. When moisture is adsorbed on the matrix, heat is released which (when $\eta > 0$) is partly taken up by the air. This will tend to increase the local temperature of the air and consequently the temperature of the air leaving the energy wheel. If H^* is positive, the warmer of the two inlet air streams is simultaneously cooled and dried and the cooler air stream is heated and humidified. When moisture is removed from the warm air by adsorption onto the matrix, energy is released and the air temperature will increase and consequently the ability for the energy wheel to cool the warm air will be reduced. The net result will be that the sensible effectiveness will decrease. On the other hand, if H^* is negative, the warm air is simultaneously cooled and humidified. In this case, when moisture is added to the warm air by desorption, the temperature of the warm air will decrease (desorption is endothermic) which will increase the sensible effectiveness of the energy wheel.

3.2.2. Cr^*_{ht}

To develop the equation for Cr^*_{ht}, the starting point is the energy equation for the matrix [equation (2)] which, upon introduction of the dimensionless independent variable for time, is

$$\frac{\rho_m Cp_m A_m}{p}\frac{\partial T_m}{\partial t^*} - h_m \frac{A_s'}{L}(\rho_v - \rho_{v,m})h_{fg}(1-\eta) - h\frac{A_s'}{L}(T_g - T_m) = 0 \quad (39)$$

where the term $\dot{m}' Cp_w(T_g - T_m)$ (which is several orders of magnitude smaller than the other terms) and axial conduction have been neglected. It should be noted that axial conduction can alter the effectiveness of energy wheels [13] which will need to be accounted for with methods described in Shah [9, 18]. Rearranging equation (39) shows that the standard dimensionless form is

$$\frac{\partial T_m}{\partial t^*} = \frac{NTU_{ht}}{Cr^*_{ht}}(T_g - T_m) \quad (40)$$

where

$$\frac{NTU_{ht}}{Cr^*_{ht}} = \frac{NTU}{Cr^*}\left[1 + \frac{h_{fg}(1-\eta)}{\rho_g Cp_g}\frac{\rho_v - \rho_{v,m}}{T_g - T_m}\right]. \quad (41)$$

Using equations (34) and (36), equation (41) can be simplified to

$$\frac{NTU_{ht}}{Cr^*_{ht}} = \frac{NTU}{Cr^*}\left[1 + (1-\eta)\frac{NTU_{ht}}{NTU_{mt}}\frac{\varepsilon_l}{\varepsilon_s}H^*\right]. \quad (42)$$

Rearranging equation (42) to solve for Cr^*_{ht} and using the definition of NTU_{ht} from equation (37) and $Le = 1$ gives

$$Cr^*_{ht} = \frac{Cr^*}{1 + \frac{\varepsilon_l}{\varepsilon_s}H^*}. \quad (43)$$

Examining equation (43) shows that Cr^*_{ht} is independent of η, but dependent on the ratio of latent to sensible effectiveness ($\varepsilon_l/\varepsilon_s$) and the operating condition factor (H^*). Since H^* varies from -6 to $+6$, in typical HVAC applications, the value of Cr^*_{ht} can be positive or negative. A negative value of Cr^*_{ht} means that, even though the supply air stream is colder than the exhaust air stream, energy will be stored in the matrix during the supply air flow due to the release of energy during phase change.

3.3. Energy transfer

There are only two independent effectiveness values for energy wheels. In this paper, the latent and sensible effectivenesses have been chosen and the total effectiveness can be derived from these. Starting with the definition of total effectiveness

$$\varepsilon_t = \frac{\dot{m}_s(H_{s,i} - H_{s,o})}{\dot{m}_{min}(H_{s,i} - H_{e,i})} = \frac{\dot{m}_e(H_{e,o} - H_{e,i})}{\dot{m}_{min}(H_{s,i} - H_{e,i})} \quad (44)$$

and introducing the definition of enthalpy as

$$H = Cp_a T_g + W(h_{fg} + Cp_v T_g) \quad (45)$$

where T is in °C, the total effectiveness can be shown to be

$$\varepsilon_t \approx \frac{\varepsilon_s + \varepsilon_l H^*}{1 + H^*}. \quad (46)$$

Equation (46) shows that ε_t is essentially a weighted average of ε_s and ε_l. As $H^* \to \infty$ (i.e., dominant moisture transfer), $\varepsilon_t \to \varepsilon_l$ and as $H^* \to 0$ (i.e., negligible moisture transfer), $\varepsilon_t \to \varepsilon_s$. Also as $H^* \to -1$, $\varepsilon_t \to \pm\infty$ if $\varepsilon_s \neq \varepsilon_l$ which was measured and simulated by Simonson et al. [11]. Equation (46) gives values of ε_t that are in close agreement with those obtained from simulations.

3.4. Saturation conditions

As a first approximation, saturation conditions can be considered to exist when a straight line between the inlet condition on the psychrometric chart cross the saturation line for cold operating conditions [19, 20]. The approximation works well for warm operating conditions and a linear sorption curve, but not for other shaped sorption curves [15, 20]. Saturation operating conditions have been addressed by Simonson and Besant [15] and Simonson et al. [20] and are not discussed here except to note why the effectiveness and governing dimensionless groups are more difficult to define under these conditions.

When the energy wheel is operating under condensation and evaporation conditions, the amount of moisture that the matrix can store is much larger than it can store during sorption conditions. This is expected to increase Cr^*_{mt} and similarly increase ε_l. Furthermore, when condensation occurs, the moisture transfer to the matrix is controlled by the mass flow rate of water vapor and the axial gradient of temperature. Condensation will occur at a rate that will keep the air, that is flowing through the wheel, saturated. This may result in a greater phase change rate than that governed by convection mass transfer with only sorption. In a sense, this is like increasing the convective mass transfer coefficient h_m. If h_m is increased, NTU_{mt} will also increase, resulting in an increase in ε_l. The value of ε_l calculated with the correlations in Part II will be a conservative estimate during condensation and frosting conditions. On the other hand, the value of ε_s calculated with the correlations in Part II will be quite accurate because ε_s is less affected by saturation conditions than ε_l [15, 20]. It is expected that the total effectiveness can still be calculated with equation (46).

4. Summary

The governing dimensionless groups for simultaneous heat and moisture transfer in energy wheels have been developed. These dimensionless groups parallel the dimensionless groups for rotary sensible heat exchangers which can be fund in many heat exchanger books. For the case of $C^* = 1$, the dimensionless groups for coupled heat and mass transfer are NTU_{mt}, Cr^*_{mt}, NTU_{ht} and Cr^*_{ht} which are related to NTU and Cr^* for rotary sensible heat exchangers. These dimensionless groups are functions of the operating temperature and humidity of the wheel and the sorption characteristics of the desiccant as well as functions of NTU, Cr^* and Crm^*. Crm^* is a moisture capacitance ratio that is analogous to Cr^*. The importance of the operating conditions are evident in the new simultaneous heat and moisture transfer dimensionless groups and an operating condition factor (H^*) has been defined which depends on the ratio of the latent to sensible energy change across the energy wheel.

To summarize the findings of Part I of this paper, the simplified governing equations for heat and moisture transfer with the dimensionless independent variables are listed below. The equations for moisture transfer are

$$\frac{\partial \rho_v}{\partial x^*} = NTU_{mt}(\rho_{v,m} - \rho_v) \tag{47}$$

and

$$\frac{\partial \rho_{v,m}}{\partial t^*} = \frac{NTU_{mt}}{Cr^*_{mt}}(\rho_v - \rho_{v,m}) \tag{48}$$

where

$$NTU_{mt} = NTU = \frac{hA_s}{\dot{m}Cp_a} \tag{49}$$

$$Cr^*_{mt} = (Crm^*)\frac{\partial u}{\partial \phi}\left(\frac{e^{\left(\frac{5294}{T}\right)}}{10^6} - 1.61\phi\right) \tag{50}$$

and

$$Crm^* = \frac{M_{d,dry}N}{\dot{m}}. \tag{51}$$

Equation (47) shows that NTU_{mt} is the proportionality constant that relates the axial gradient of water vapor density in the air to the driving potential for moisture transfer which is the difference between the water vapor density on the surface of the desiccant to the water vapor density in the air. Similarly, equation (48) shows that NTU_{mt}/Cr^*_{mt} is the proportionality constant which relates the storage of water vapor in the desiccant to the potential for moisture transfer. These proportionality constants are clearly dependent on the physical size, shape and rotational speed of the energy wheel as well as the mass flow rate of air. Cr^*_{mt} also depends on the operating temperature and humidity. It should be noted that the definition of NTU presented in equation (49) uses the approximation that $(\dot{m}Cp)_g \approx \dot{m}Cp_a$ for typical operating conditions for energy wheels.

The simplified equations for heat transfer are

$$\frac{\partial T_g}{\partial x^*} = NTU_{ht}(T_m - T_g) \tag{52}$$

and

$$\frac{\partial T_m}{\partial t^*} = \frac{NTU_{ht}}{Cr^*_{ht}}(T_g - T_m) \tag{53}$$

where

$$NTU_{ht} = \frac{NTU}{1 + \eta\frac{\varepsilon_l}{\varepsilon_s}H^*} \tag{54}$$

$$Cr^*_{ht} = \frac{Cr^*}{1 + \frac{\varepsilon_l}{\varepsilon_s}H^*} \tag{55}$$

and

$$Cr^* = \frac{(MCp)_m N}{\dot{m}Cp_a}. \tag{56}$$

Equations (52) and (53) are analogous to equations (47) and (48) except that Cr^*_{ht} (and NTU_{ht}, if η is large) can have negative values. This may seem physically impossible, but negative values for NTU_{ht} and Cr^*_{ht} can be explained in light of the governing equations shown above. For example, equation (52) shows that the axial gradient of the temperature in the air will be proportional to the temperature difference between the air and the matrix. The proportionality constant, NTU_{ht}, will typically be positive. However, if the moisture transfer is dominant and in the opposite direction of the heat transfer (i.e., H^* is a large negative value), NTU_{ht} can have a negative value. According to equation (52) this means that the axial temperature gradient in the air will be negative when $T_m > T_g$ and NTU_{ht} is negative. This is an important observation because it means that even though the matrix is warmer than the air, the air temperature can decrease as it flows through the wheel if the heat released due to phase change is large.

Similarly in equation (53), the change in temperature of the matrix with time is proportional to the temperature difference between the gas and matrix. Under normal conditions, the temperature of the matrix will increase when the temperature of the air is warmer than the temperature of the matrix. However, in the case of $NTU_{ht}/Cr^*_{ht} < 0$, the change in the temperature of the matrix will be opposite in the convection heat transfer. That is, when the moisture transfer is large and in the opposite direction of the heat transfer, the temperature in the matrix can increase with time even though the temperature of the matrix is greater than the temperature of the air. This means that the heat transfer due to moisture transfer is larger than the heat transfer due to radial convection, resulting in the net heat transfer to be in the opposite direction of the simple radial convection. These observations are very important because they explain the coupling between the heat and moisture transfer.

Finally, it is important to note that the assumption of $Le = 1$ is used in this analysis. As shown previously, this means that $NTU_{mt} = NTU$ but does not guarantee that $\varepsilon_s = \varepsilon_l$ because NTU_{ht}, Cr^*_{ht} and Cr^*_{mt} can all have different values depending on the wheel design and operating conditions.

Summarizing, the effectiveness of energy wheels for $C^* = 1$, $0.25 \leqslant (hA_s)^* \leqslant 4$ and $k_m = 0$ is expected to be

$$\varepsilon_s = f_1(NTU_{ht,o}, Cr^*_{ht,o}) \quad (57)$$

$$\varepsilon_l = f_2(NTU_{mt,o}, Cr^*_{mt,o}) \quad (58)$$

and

$$\varepsilon_t = f_3(\varepsilon_s, \varepsilon_l, H^*) \quad (59)$$

where the overall dimensionless groups are defined as

$$NTU_{mt,o} = NTU_o \quad (60)$$

$$Cr^*_{mt,o} = (Crm^*_o)\frac{\partial u}{\partial \phi}\left(\frac{e^{\left(\frac{5294}{T}\right)}}{10^6} - 1.61\phi\right) \quad (61)$$

$$NTU_{ht,o} = \frac{NTU_o}{1+\eta\frac{\varepsilon_l}{\varepsilon_s}H^*} \quad (62)$$

and

$$Cr^*_{ht,o} = \frac{Cr^*_o}{1+\frac{\varepsilon_l}{\varepsilon_s}H^*} \quad (63)$$

with

$$NTU_o = \frac{1}{(\dot{m}Cp_a)_{min}}\left[\frac{1}{(hA_s)_s} + \frac{1}{(hA_s)_e}\right]^{-1} \quad (64)$$

$$Cr^*_o = \frac{(MCp)_m N}{(\dot{m}Cp_a)_{min}} \quad (65)$$

$$Crm^*_o = \frac{M_{d,dry}N}{\dot{m}_{min}} \quad (66)$$

and

$$H^* = \frac{\Delta H_l}{\Delta H_s} = \left(\frac{\Delta H_s}{\Delta H_t}\right)^{-1} - 1 = 2500\frac{\Delta W}{\Delta T}. \quad (67)$$

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References

[1] K. Rengarajan, D.B. Shirey III, R.A. Raustad, Cost-effective HVAC technologies to meet ASHRAE Standard 62-1989 in hot and humid climates, ASHRAE Trans. 102(1) (1996) 166–182.

[2] D.B. Shirey III, K. Rengarajan, Impact of ASHRAE Standard 62-1989 on small Florida offices, ASHRAE Trans. 102(1) (1996) 153–165.

[3] D.R. Irwin, C.J. Simonson, K.Y. Saw, R.W. Besant, Contaminant and heat removal effectiveness and air-to-air heat/energy recovery for a contaminated air space, ASHRAE Trans. 104(2) (1998).

[4] J.A. Schaefer, R. Diblasio, E.J. Bernacki, The effectiveness of a molecular sieve heat wheel in maintaining good air quality in a major research facility, in: K.Y. Teichman (Ed.), Proceedings of IAQ 96—Paths to Better Building Environments, ASHRAE, Atlanta, 1996, pp. 223–225.

[5] C.C. Downing, C.W. Bayer, Classroom indoor air quality vs. ventilation rate, ASHRAE Trans. 99(2) (1993) 1099–1103.

[6] R.K. Shah, E.K. Subbarao, R.A. Mashelkar, Heat Transfer Equipment Design, Hemisphere, New York, 1988.

[7] S. Kakaç, A.E. Bergles, F. Mayinger, Heat Exchangers: Thermal-Hydraulic Fundamentals and Design, Hemisphere, New York, 1981.

[8] J.E. Coppage, A.L. London, The periodic-flow regenerator—a summary of design theory, Trans. ASME 75 (1953) 779.

[9] R.K. Shah, Thermal design theory for regenerators, in: S. Kakaç, A.E. Bergles, F. Mayinger (Eds.), Heat Exchangers: Thermal-Hydraulic Fundamentals and Design, Hemisphere, New York, 1981, pp. 721–763.

[10] W.M. Kays, A.L. London, Compact Heat Exchangers, McGraw-Hill, Toronto, 1984.

[11] C.J. Simonson, D.L. Ciepliski, R.W. Besant, Determining the performance of energy wheels: Part II—experimental data and numerical validation, ASHRAE Trans. 105(1) (1998).

[12] C.J. Simonson, R.W. Besant, Heat and moisture transfer in desiccant coated rotary energy exchangers: Part I—numerical model, Int. J. HVAC&R Research 3(4) (1997) 325–350.

[13] C.J. Simonson, R.W. Besant, Heat and moisture transfer in desiccant coated rotary energy exchangers: Part II—validation and sensitivity studies, Int. J. HVAC&R Research 3(4) (1997) 351–368.

[14] G. Stiesch, S.A. Klein, J.W. Mitchell, Performance of rotary heat and mass exchangers, Int. J. HVAC&R Research 1(4) (1995) 308–323.

[15] C.J. Simonson, R.W. Besant, Heat and moisture transfer in energy wheels during sorption, condensation and frosting conditions, ASME J. Heat Transfer 120(3) (1998) 699–708.

[16] T.J. Lambertson, Performance factors of a periodic-flow heat exchanger, Trans. ASME 80(1) (1958) 586–592.

[17] G.D. Bahnke, C.P. Howard, The effect of longitudinal heat conduction on periodic-flow heat exchangers, Trans. ASME, Journal of Engineering for Power 86 (1964) 105–120.

[18] R.K. Shah, Counter flow rotary regenerator thermal design procedures, in: R.K. Shah, E.K. Subbarao, R.A. Mashelkar (Eds.), Heat Transfer Equipment Design, Hemisphere, New York, 1988, pp. 267–296.

[19] R.B. Holmberg, Prediction of condensation and frosting limits in rotary wheels for heat recovery in buildings, ASHRAE Trans. 95(1) (1989) 64–69.

[20] C.J. Simonson, R.W. Besant, G.W. Wilson, Condensation and frosting in energy wheels, in: K. Vafai, J.L.S. Chen (Eds.), Proceedings ASME 32nd National Heat Transfer Conference, vol. HTD 339(1), Current Developments in Numerical Simulation of Heat and Mass Transfer, ASME, New York, 1997, pp. 161–169.